Extraordinary broadband impedance matching in highly dispersive media - the white light cavity approach

JACOB SCHEUER,1,2,* DMITRY FILONOVO,3 TATYANA VOSHEVA,3 AND PAVEL GINZBURG1

1School of Electrical Engineering, Tel Aviv University, 69978 Tel Aviv, Israel
2Center for Light-Matter Interactions, Tel Aviv University, 69978 Tel Aviv, Israel
3Center of Photonics and 2D materials, Moscow Institute of Physics and Technology, Dolgoprudny 141700, Russia

*ko@eng.tau.ac.il

Abstract: Suppressing reflections from material boundaries has always been an objective, common to many disciplines, where wave phenomena play a role. While impedance difference between materials necessarily leads to a wave reflection, introducing matching elements can almost completely suppress this phenomenon. However, many impedance matching approaches are based on resonant conditions, which come at a price of narrow bandwidth operation. Although various impedance matching architectures have been developed in the past, many of them fail to produce a broadband and flat (ripple-free) transmission, particularly in the presence of strong chromatic dispersion. Here we propose and demonstrate an approach for designing an optimal matching stack capable of providing a flat broadband transmission even in the presence of significant group velocity dispersion. As an experimental example for the method verification, we used a strong modal dispersion in a rectangular waveguide, operating close to a mode cut-off. The waveguide core consists of alternating polymer sections with a variable filling factor, realized using additive manufacturing. As a result, a broadband matching in the range of 7-8GHz was demonstrated and proved to significantly outperform the standard binomial transformer solution. The proposed method can find use across different disciplines, including optics, acoustics and wireless communications, where undesired reflections can significantly degrade system’s performances.

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1. Introduction

Partial reflection occurs when a wave impinges upon an interface between two media with different impedances. This fundamental phenomenon is common to all types of waves – electromagnetic, acoustic and as well as to the wave-functions of quantum particles [1,2]. Reducing reflectivity, i.e. obtaining impedance matching or “anti-reflection”, and maximizing transmission is crucial in designing various opto-electronic and acoustic devices [3,4]. In optics and radio frequencies (RF), enhancing power transmission is essential for obtaining high-efficiency solar cells and detectors [3,5–8], glare-free displays and touchscreens [9], imaging, augmented reality, telecommunications and many more [10]. In acoustics, impedance matching is important for the realization of high quality absorbers, sonars, ultra-sound transducers and medical ultrasonic sensors [11–14]. Anti-reflection structures are also observed in nature, e.g. the reflection-less cornea of nocturnal moths, which disguise these insects from potential predators [15,16].

Since the pioneering works of Lord Raleigh [17] and Fraunhofer [18] who showed that reflection from an interface can be suppressed by an intermediate layer with graded index, a variety of approaches to realize anti-reflection coating has been proposed and demonstrated. These approaches included thin film anti-reflection coatings [3,19,20], adiabatic graded index
layers [21–26], resonant nanostructures [1-, 27], and metamaterials [28]. Although 3D graded index layers and metasurfaces can provide a broadband, omni-directional and polarization insensitive impedance matching [10,25,27,28], the design and fabrication of such anti-reflection components are complex and expensive. Consequently, the most commonly used anti-reflection measures are based on thin films (in optics) or cascaded resonators (in RF) [29–32] designed to reduce reflection by generating destructive interference of the partial reflections from the interfaces separating the layers. The simplest antireflection coating is the quarter-wavelength transformer: a section with a length of a quarter of the wavelength (at which the reflection is minimized) and impedance, which is the geometric average of the impedances of the input and the load media - $Z_{\lambda/4} = \sqrt{Z_{\text{in}}Z_{\text{load}}}$. The combination of the section (or film) length and the impedance $Z_{\lambda/4}$ makes the transformer a critically coupled resonator, which yields complete transmission (no reflection) on resonance (see supplemental document for details).

The single quarter-wavelength transformer, which is the most commonly used approach for obtaining impedance matching, exhibits relatively narrow bandwidth by design. The line-shape of the transmission spectrum for the high-Q case is Lorentzian, which introduces phase and amplitude distortions into broadband signals. This limitation might be severe in practical applications, such as high-speed telecommunication. In order obtain a broader and flatter spectral response, more sophisticated anti-reflection schemes comprising multiple layers/sections are required. Such multi-section impedance matching approach requires an intensive optimization for obtaining sections lengths and corresponding impedances. However, brute-force optimization provides generic electromagnetic parameters, which do not necessarily comply with an experimental reality. For example, a so-called optimal matching necessitates using sections with specific impedances, which can be hardly realizable in practice. The reason is that arbitrary refractive indices are not easily obtainable because of the limited number of naturally available materials [33].

In many cases, however, optimal performance can be traded for other application-specific requirements. For example, maximally flat (i.e. ripple-free) transmission bands are desired in high-speed communications. These requirements can be delivered by Binomial impedance transformers [34], which are designed to set the first $N-1$ derivatives of the frequency response at the matching frequency to zero. $N$ corresponds to the number of layers/sections comprising the transformer. The spectral reflection of such a transformer is given by $\Gamma = A \cdot (1 - e^{-i\pi\Delta f/f_0})^N$ where $f_0$ is the matching frequency and $\Delta f$ is the frequency detuning from $f_0$. Obtaining this shape provides a set of equations, which determines the required partial reflectivities between the sections and, hence, the impedances of the individual transformer sections. Although Binomial transformers seem to provide the desired maximally flat impedance matching, their design assumes implicitly that the transformer sections are either dispersion-less or that the dispersive properties of all sections are identical. This assumption is necessary in order to describe the reflection spectrum from the transformer as a polynomial in the frequency detuning. However, in realistic structures, where both material and modal dispersions play a role, the aforementioned approach becomes problematic. For example, in waveguides close to a cutoff point, this assumption is strictly invalid and obtaining flat, broadband, impedance matching based on Binomial approach, fails.

In this paper, we present and demonstrate a new approach for obtaining maximally-flat and broadband impedance matching by constructing a critically coupled white light cavity (WLC) – a cavity with unique dispersive properties that extend its resonance over a spectral band [35]. This approach facilitates obtaining the required impedances of the transformer section without restrictions or assumption on their dispersive properties. As a specific example, we demonstrate broadband impedance matching in a dispersive metallic waveguides platform in the microwave range. We show that our approach outperforms the well-established Binomial transformer approach, yielding low, broadband and flat reflection.
2. White light cavity – concept and design

As noted in the introduction, impedance matching can be obtained at the resonance frequencies of a critically coupled cavity and will result in suppressing reflections only over a discrete set of frequencies. However, if the cavity is made to resonate over a continuous spectral range, impedance matching could be obtained over a broader frequency band. Such an approach is known as a white light cavity (WLC). The concept of WLC was proposed by Wicht [35] and later demonstrated by the groups of Shahriar [36,37] and Smith [38,39]. The idea is to introduce into the cavity an additional element, which exhibit a negative group velocity ($n_g < 0$). This additional element compensates the propagation phase and, as a result, the resonant condition (overall phase $= 2\pi m$, $m \in \mathbb{N}$) is satisfied over a continuous spectral band. This concept is illustrated in Fig. 1: a Fabry-Perot (FP) cavity which includes an internal phase component with phase response $\theta(\omega)$. When the WLC condition is satisfied, the roundtrip phase in the cavity becomes flat (i.e. frequency independent), thus leading to a broader resonance condition. This is obtained when $d\theta/d\omega = -2L_{cav}d\beta/d\omega$, where $L_{cav}$ and $\beta$ are the respectively the cavity length and the wavenumber. A detailed description of the WLC concept is given in the supplemental document.

Fig. 1. (a) Schematics of a regular cavity transmission – high transmissivity is obtained only for a narrow band spectrum. (b) Schematics of a white light cavity transmission - high transmissivity is obtained for a broadband spectrum owing to an introduction of a dispersion compensation layer. (c) Schematics of a low reflectivity spectra – narrow band (regular, Black) versus broadband (WLC, red) responses. (d) Dispersion properties of the cavity and the compensating layer – the opposite slopes cancel each other, granting a flat broadband response.

The key ingredient in forming a WLC is the intra-cavity component providing the negative phase slope. As shown in [40], a WLC can be realized a structure comprising two coupled resonators, where one is considered as the “main” cavity, while the other is the phase component. The required coupling coefficients between the cavities and the input/output (I/O) waveguides are determined by the WLC condition [40] and by the requirement for critical coupling. Figure 2 depicts a schematic of the WLC impedance transformer. The left section is considered as the “main” cavity, while the adjutant one, on the right-hand side, is the phase component. The length of each cavity is $\lambda/4$ ($\lambda$ here corresponds to the wavelength in the structure) in order to resonate at the matching frequency $\omega_0$ (note that it is assumed that $Z_{in} > Z_1 > Z_2 > Z_{Load}$).

The critical coupling of the “main” cavity is achieved when $|r_L| = |r_R^{\text{eff}}|$, where $r_L = (Z_{in} - Z_1)/(Z_{in} + Z_1)$. $r_R^{\text{eff}}$ is the effective reflectivity due to the right FP cavity:

$$r_R^{\text{eff}} = \frac{-r_{21} + r_{2L}e^{-i\theta_2}}{1 - r_{21}r_{2L}e^{-i\theta_2}} = |r_R^{\text{eff}}|e^{-i\theta}, \quad (1)$$
where $\phi_2$ is the roundtrip phase in the right FP cavity, $r_{21} = (Z_1 - Z_2)/(Z_1 + Z_2)$ and $r_{2L} = (Z_{\text{Load}} - Z_2)/(Z_{\text{Load}} + Z_2)$. On the first resonance, $\phi_2 = \pi$ and the critical coupling condition leads to the following relation between $Z_1$ and $Z_2$:

$$Z_2^2 Z_{\text{in}} = Z_1^2 Z_{\text{Load}}$$  \hfill (2)

To attain the WLC condition we require [40]:

$$\frac{d\theta}{d\omega} \bigg|_{\omega=\omega_0} = \frac{r_{21}^2 (1 - r_{21}^2)}{(r_{21} + r_{2L})(1 - r_{21}r_{2L})} \cdot \frac{d\phi_2}{d\omega} \bigg|_{\omega=\omega_0} = -2L_1 \frac{d\beta_1}{d\omega} \bigg|_{\omega=\omega_0}$$  \hfill (3)

where $\phi_2$ is the roundtrip phase in section 2 in Fig. 2 and $L_1$, $\beta_1$ and $L_2$, $\beta_2$ are length and propagation coefficients in sections 1 and 2 respectively. Note, that if the dispersion (material and modal) in the section is negligible, then the derivatives of the roundtrip phase in section $i$ with respect to the frequency is simply a constant: $d\phi_i/d\omega = 2L_i n_i^{\text{eff}}/c = 2L_i \mu_0/Z_i$. In this case, the combination of the WLC and the critical coupling conditions yields:

$$Z_2^2 = \sqrt{Z_3^2 Z_{\text{Load}}}; \quad Z_1^2 = \sqrt{Z_{\text{in}}^2 Z_{\text{Load}}},$$  \hfill (4)

which is precisely a two-section Binomial impedance transformer [34]. However, here we utilize a rectangular metallic waveguide operating at its lower guided mode (TE$_{01}$), where the propagation coefficient and characteristic impedance are given by:

$$\beta_{\text{TE}_{01}} = k_0 \sqrt{\varepsilon_r - (\lambda/2a)^2}; \quad Z = \frac{\eta_0}{\sqrt{\varepsilon_r - (\lambda/2a)^2}},$$  \hfill (5)

where $k_0$ and $\lambda$ are respectively the wavenumber and wavelength in vacuum, $\varepsilon_r$ is the dielectric coefficient of the material filling the waveguide, $\eta_0$ is the vacuum wave impedance and $a$ is the waveguide width (for the lowest-order mode the short axis of the cross section does not matter). The dispersion of the mode propagation constant is governed by the confining structure, as the material dispersion is negligible (see supplemental document). In this case, the WLC condition is given by:

$$\frac{Z_1 Z_2 (Z_{\text{Load}}^2 - Z_2^2)}{Z_{\text{Load}}^2 Z_1^2 - Z_2^4} \cdot \frac{\varepsilon_{r,2}}{\beta_2^2} + \frac{\varepsilon_{r,1}}{\beta_1^2} = 0,$$  \hfill (6)

where $\varepsilon_{r,i}$ and $\beta_i$ are respectively the dielectric coefficient and propagation coefficients in section $i$. Note, that $\beta_i$ and $Z_i$ are all determined by $\varepsilon_{r,i}$. Consequently, solving Eqs. (6) and (2) yields the required $\varepsilon_{r,1}$ and $\varepsilon_{r,2}$ needed for obtaining the maximally flat impedance matching structure.
3. Matching a dispersive waveguide - design

As a concrete example, consider a rectangular metallic waveguides operating at its lower order mode. The dimensions of the waveguide are $a = 22.86\text{mm}$, $b = 10.16\text{mm}$. We would like to impedance match an empty waveguide ($\varepsilon_{r,1} = 1$) to a waveguide with the same dimensions which is filled with a material with a dielectric constant $\varepsilon_{r,2} = 2.526$ using a double section transformer. The choice of dimensions and material parameters is set according to the experiments described below. The cutoff frequency of the TE$_{01}$ mode in the empty waveguide is 6.56GHz. In order for the waveguide dispersion to be non-negligible we choose to impedance match the waveguides relatively close (but not too close) to the cut-off frequency, i.e. in the vicinity of $f = 7.5\text{GHz}$.

According to the structure and material parameters the impedances of the input and load waveguides are $Z_{in} = 777.6\Omega$ and $Z_{Load} = 283.5\Omega$. Using Eqs. (2) and (6), which are solved numerically, we obtain $\varepsilon_{r,1} = 1.10308 (Z_1 = 648.2\Omega)$ and $\varepsilon_{r,2} = 1.69053 (Z_2 = 391.6\Omega)$. The length of each section (quarter wavelength) is given by $L_i = \pi/2\beta_i = \frac{\lambda}{4Z_i/\eta_0}$. As a comparison, the classical (i.e. dispersion-less) Binomial transformer approach (Eq. (4)) yields $\varepsilon_{r,1} = 1.1537 (Z_1 = 604.4\Omega)$ and $\varepsilon_{r,2} = 1.8292 (Z_2 = 365.2\Omega)$. Note, that the lengths of the section also differ from that of the optimal design. Figure 3 compares the calculated power reflectivity, calculated using transfer matrix method, for the two designs described above. It is clear that the optimal design based on the WLC approach provides a flatter and ripple-free impedance matching over a broader spectral range compared to that of the classical Binomial transformer. The reflection from a single section (quarter-wavelength) transformer is also plotted in Fig. 3, demonstrating the single frequency matching and a failure to provide a broadband response.

![Graph showing comparison between reflectivities of different transformers](image)

**Fig. 3.** Comparison between the reflectivities of conventional dual section Binomial transformer (blue), WLC (red) and a l/4 transformer (dashed black). $|S_{11}|^2$ – reflectivity or the field intensity.

4. Matching a dispersive waveguide – experimental demonstration

To verify the concept, we use the WLC approach for obtaining broadband impedance matching between two metallic rectangular waveguides. The choice for the frequency band is inspired by several practical considerations. It can be seen that permitivities of matching layers have to be quite accurate. Advanced additive manufacturing (3D printing) allows fabricating volumetric samples with a controllable filling factor, i.e. the volume of plastic can be adjusted. Mixing between polymer filaments and air voids allows tuning material properties quite accurately and predict the results with simple volume-averaged mixing formulas (e.g. [41]).
Considering the aforementioned approaches and using calibrated waveguide section (X-band waveguide), the following parameters have been chosen: the waveguide cross-section is 22.86mm X 10.16mm and it is filled with a low-loss, polymer material (ABS), whose permittivity ($\varepsilon_r$) can be varied between 1 (a complete void) and 2.53 (a solid plastic bulk). The dielectric coefficient can be modified between these extreme values by controlling the density of the material. The central frequency of the impedance matching is set to 7.5GHz. Figure S3 depicts the measured dielectric coefficient of the plastic, as a function of the filling factor at two representative frequencies - 8.2GHz and 12.4GHz. The flexibility of additive manufacturing allows tailoring dielectric constant almost on demand, making this method and the corresponding frequency range attractive for future practical applications.

The characterization of the impedance matching transformer was carried out by simple measurement of the transmission/reflection coefficients with an RF network analyzer. However, as assessing fields inside a material is difficult, a symmetric stack, comprising two impedance matching transformers was characterized: from an empty section to 100% plastic filled and then back to an empty waveguide (inset of Fig. 4).

![Figure 4](image_url)

**Fig. 4.** Experimental reflectivity spectra of a symmetric stack – classical (binomial) and optimal (our WLC approach). Inset – a photograph of the optimal design sections.

Figure 4 depicts a comparison of the measured $|S_{11}|^2$ parameters, obtained for the classical Binomial transformer and our WLC approach. The reflection from the WLC shows a broader and flatter response in respect to the Binominal design. Note that the reflectivity level does not reach 0. This is attributed to the losses in the material (which are neglected in the calculations) as well as to small deviation from the optimal dielectric coefficient and sections dimensions. The most noticeable feature of the WLC based design is the flatness and the literally ripple-free nature of the impedance matching over $\sim$1GHz bandwidth. On the other hand, the Binomial transformer exhibits an asymmetric spectral response and a non-negligible $\sim$5% ripple over a similar bandwidth.

In the design of the WLC based impedance transformer discussed above, the dominating dispersion effect originates from the waveguide structure. Consequently, the WLC condition, Eq. (6), can be obtained analytically if the waveguide impedance and propagation coefficient on the frequency is known (Eq. (5)). Nevertheless, material dispersion (if present) can be incorporated as well (see supplemental document), leading to a small modification of Eq. (6). The WLC approach can be extended to the design of broader and flatter impedance matching transformers incorporating more quarter-wavelength sections. To obtain the dielectric coefficients for an impedance matching transformer comprising $N$ sections, the WLC condition is extended to require that the first $N-1$ derivatives of the roundtrip phase are zero (see the supplemental document for an example for $N = 3$). Nevertheless, it should be noted that the WLC design
approach is useful for obtaining maximally-flat (i.e. Butterworth filter-like) impedance matching. It is less useful for obtaining Chebyshev filters-like response, which provides broader bandwidth and steeper roll-off at the cost of a larger ripple.

5. Conclusions

To conclude, we have proposed and demonstrated a new approach for broadband impedance matching that is capable of addressing the impact of dispersion. The design approach originates from the physical understanding of the requirements for broadband and ripple-free transmission: a critically coupled WLC formed by a multi-section structure connecting between the input and the load waveguide. The approach can be straightforwardly extended to the design of broader and flatter impedance matching transformers comprising more quarter-wavelength sections. The WLC effect extends the bandwidth of the resonator while the critical coupling ensures that only a small fraction of the impinging power is reflected. This design approach is universal and can be applied for broadband anti-reflection coating, coupled-resonator filters, perfect absorption and more.

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Supplemental document. See Supplement 1 for supporting content.

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