

# Low-frequency nonlocal and hyperbolic modes in corrugated wire metamaterials

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Abstract: Metamaterials based on arrays of aligned plasmonic nanowires have recently attracted significant attention due to their unique optical properties that combine tunable strong anisotropy and nonlocality. These optical responses provide a platform for implementation of novel sensing, imaging, and quantum optics applications. Basic building blocks, used for construction of those peculiar composites, are plasmonic metals, such as gold and silver, which have moderate negative values of permittivities at the optical spectral range. Scaling the plasmonic behavior to lower frequencies remains a longstanding challenge also owing to the emergence of strong spatial dispersion in homogenized artificial composites. At lower THz and GHz frequencies, the electromagnetic response of noble metals approaches that of perfect electric conductors, preventing straightforward scaling of visible-frequency plasmonics to the frequency domains that are important for a vast range of applications, including wireless communications, microwave technologies and many others. Here we demonstrate that both extreme anisotropy (so-called hyperbolicity) and nonlocality of artificial composites can be achieved and designed in arrays of corrugated perfectly conducting wires at relatively low GHz frequencies. The key concept is based on hybridization of spoof plasmon polariton modes that in turn emulate surface polariton waves in systems with corrugated interfaces. The method makes it possible to map the recent developments in the field of plasmonics and metamaterials to the domain of THz and RF photonics.

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### 1. Introduction

Plasmonics is a rapidly developing field of nanophotonics focusing on optical phenomena with noble metals. These materials, having negative permittivities at optical and infrared (IR) spectral ranges, introduce frontier possibilities in tailoring and designing peculiar electromagnetic interactions between light and matter, which can be utilized in a range of practical applications. Apart from intriguing responses of individual metal elements, structured arrays of plasmonic components form a flexible platform for metamaterials - composite media with engineered optical responses unavailable in nature. In particular, metamaterials formed by arrays of aligned plasmonic nanowires grown in dielectric matrices operating at visible and near-infrared frequencies, have been recently used to demonstrate super resolution imaging [1], and super-absorbers [2], achieve record-high performances of bio- and acousto-optical sensors [3,4], to enable novel nonlinear optical platforms [5], realize negative refraction of light [6], achieve optical attraction forces [7], and record-enhancement of density of states [8]. These phenomena are enabled by extreme optical anisotropy (also known as hyperbolicity) in combination with strong nonlocality of nanowire composites.

Unfortunately, due to materials dispersion, advances in visible-frequency plasmonics can be rarely reproduced at lower, GHz and THz frequencies. Attempts to mimic electron plasma behavior at GHz frequencies with arrays of ultra-thin wires [9] faced challenges even at extremely long wavelength limit [10,11]. Nevertheless, the behavior of individual surface plasmon polariton waves, propagating along a metal-dielectric surface, can be mimicked with spoof plasmons, electromagnetic modes supported by structured perfectly conducting interfaces [12–14].

Here we propose a novel metamaterial platform that relies on hybridized spoof plasmons to realize hyperbolic and nonlocal modes in low-frequency systems. We analyze numerically the dispersion of these modes and demonstrate the analytical mapping between the low-frequency response of corrugated wire systems and the formalism, proposed in Ref [15]. by Wells et.al. for high-frequency response of plasmonic nanowire media. The rest of the manuscript is organized as follows: Sections 2 and 3 introduce important concepts of the nonlocal plasmonic wire media and of spoof plasmons and provides a brief review of relevant recent results. Section 4 presents the spoof wire media and outlines the results of exact numerical solutions of Maxwell equations. in these materials. The analytical mapping between optical response of plasmonic wires and low-frequency behavior of spoof wire metamaterials is presented in Section 5. Section 6 concludes the manuscript.

## 2. Nonlocal nanowire metamaterials in the optical domain

Optical response of metals is dominated by the dynamics of their free electrons. At visible and near-IR frequencies, this plasma-like dynamics causes permittivity of metals to become negative, enabling excitation of sub-wavelength plasmons resulting from coupling between free-space photons and electrons. Photon-plasmon interaction enable a special optical mode, known as surface plasmon polariton, a guided wave that propagates at the boundary between plasmonic and dielectric media and exponentially decays away from the interfaces into both of surrounding materials [16].

When multiple structures supporting surface plasmon polaritons are brought to close proximity to each other, individual polariton modes hybridize with each other, often enabling new types of guided waves [17]. Specifically, in metamaterial composites formed by arrays of aligned plasmonic wires (Fig. 1), hybridization of cylindrical surface plasmons supported by individual metallic yields the formation of bulk modes. The dispersion of the resulting optical modes can be described in terms of effective permittivity tensor, whose diagonal components  $\hat{\epsilon} = \{\epsilon_{\perp}, \epsilon_{\perp}, \epsilon_{z}\}$  are given by:

$$\epsilon_{\perp} = \epsilon_{\perp}^{mg}; \epsilon_{z} \left( k_{z} \right) = \xi \left[ k_{z}^{2} - \left( k_{z}^{l} \right)^{2} \right] \frac{c^{2}}{\omega^{2}}$$
(1)

where  $\hat{\epsilon}^{mg}$  represent the components of the effective permittivity obtained in quasi-static limit using Maxwell Garnett formalism [18],

$$\epsilon_{\perp}^{mg} = \epsilon_d \frac{(1+p)\epsilon_m + (1-p)\epsilon_d}{(1+p)\epsilon_d + (1-p)\epsilon_m}; \ \epsilon_z^{mg} = p\epsilon_m + (1-p)\epsilon_d \tag{2}$$

*p* represents the volume-fraction of the wires in the system,  $\epsilon_m$  and  $\epsilon_d$  represent the permittivity of the metal wires and dielectric matrix,  $k_z^l$  represents the wavenumber of the collective plasmon-polariton mode propagating along the nanowires and the parameter  $\xi$  can

be determined via  $\xi = p \frac{\epsilon_m + \epsilon_d}{\epsilon_d - n_{\infty}^2}$ , with  $n_{\infty} = \lim_{\epsilon_m \to -\infty} k_z^l c / \omega$ .

Note that the component of the permittivity in the direction along the wires is nonlocal (it explicitly depends on the wavevector). As result of nonlocality, the composite supports three waves, one TE-polarized "ordinary" wave and two TM-polarized "extraordinary" modes [or one TM and one longitudinal (additional) wave]. Explicitly, dispersion of the TE wave is given by:

$$k_x^2 + k_z^2 = \epsilon_\perp \frac{\omega^2}{c^2}$$
(3a)

and the dispersion of the extraordinary modes is given by

$$\frac{k_x^2}{\epsilon_z(k_z)} + \frac{k_z^2}{\epsilon_\perp} = \frac{\omega^2}{c^2}$$
(3b)

When  $k_x = 0$  and the modes propagate parallel to optical axis of metamaterial, the dispersion of one of extraordinary modes is identical to that of TE wave; the second extraordinary mode, formally described by  $\epsilon_{zz}(k_z) = 0$ , represents longitudinal-like electromagnetic wave.

The dispersion of optical modes in plasmonic nanowires depend on multiple (often, interdependent) parameters including wavelength, material permittivity, and geometry. However, regardless of the exact parameter variation, this dispersion exhibits universal

behavior. To illustrate this behavior, we follow the approach of Ref [15]. and consider the hypothetical situation when the geometry of metamaterial and excitation wavelength are fixed and vary relative permittivity of wires.

The effective indices and the field distributions of transverse and longitudinal modes supported by the wire composites are summarized in Fig. 1. It is seen that the transverse waves represent oscillations of electron plasma perpendicular to wires, while longitudinal mode represents plasmonic oscillations along the nanowires. Note that since the polarizations of the two types of modes are orthogonal to each other, their dispersions are allowed to intersect at some point on the diagram [no anti-crossing phenomenon is observed in Fig. 1(c)].

However, when  $k_x \neq 0$ , "transverse" oscillations of electrons do couple to their "longitudinal" counterparts. As result of this coupling, the dispersion of the two TM modes exhibit a typical avoided crossing ehavior with wavenumber  $k_x$  playing the role of the effective coupling strength. Therefore, as the coupling strength is increased, the (square of) one of the effective modal indices grows and the other one decays. Mathematically [see Eq. (3)b)], evolution of the first mode can be mapped to hyperbola in  $k_x$ ,  $k_z$  space, while the behavior of second mode is described by an ellipse. Hyperbolic-like modes, characterized by growing  $k_z(k_x)$  behavior, has been utilized to postpone the onset of diffraction limit, to engineer optical density of states, and novel type of nonlinearities [2–8,19].



Fig. 1. (a) Schematic of the plasmonic nanowire composite;  $a = 100nm, r = 20nm, \epsilon_d = 1$ ; vacuum wavelength  $\lambda_0 = 1.5\mu m$ ; (b) effective medium parameters, according to the Maxwell-Garnett approximation; (c) effective modal index of the transverse (orange,  $k_z c / \omega \approx 1$ ) and strongly dispersive longitudinal (red, blue) modes propagating parallel to the wires; panel (d) illustrates dispersion of the TM (red, blue lines) and TE (orange) modes propagating obliquely  $[k_x = 0.3\omega/c]$  to the wires; panels (e,f,g) illustrate the distribution of electric field in the unit cell of the TE, TM, and longitudinal modes, respectively; in the limit  $k_x \rightarrow 0$  propagation of transverse mode converges to predictions of Maxwell-Garnett effective medium theory [15]

Interestingly, effective out-of-plane permittivity of the "transverse" extraordinary wave in the limit  $k_x \rightarrow 0$  converges to  $\epsilon_{zz}^{mg}$  [15]. This wave has elliptic behavior at relatively high (visible) frequencies where  $\epsilon_{zz}^{mg} > 0$ , and switches to hyperbolic response at lower frequencies where  $\epsilon_{zz}^{mg} < 0$ . The "additional" extraordinary wave operates in hyperbolic regime when  $\epsilon_{zz}^{mg} > 0, \epsilon_m < -\epsilon_d$  and becomes exponentially decaying mode when  $\epsilon_{zz}^{mg} < 0$ . At ultra-low frequencies where  $|\epsilon_m| \gg 1$ , electromagnetic response of metamaterial approaches epsilon-near-infinity limit that can be used for high-resolution imaging [1] but that does not provide the benefits of modulation of density of optical states associated with nonlocal epsilon-near-zero or hyperbolic systems [19].

# 3. Spoof plasmons

The concept of spoof plasmon was first introduced in Ref [12]. by Pendry et.al. In contrast to surface plasmon polaritons that propagate at the smooth metal-dielectric interface, spoof plasmons propagate at the interface between a dielectric and structured perfectly conducting metal and can be thought to result from coupling between cavity modes supported by the individual corrugations [14]. Similar to surface plasmon polaritons, electromagnetic fields in spoof plasmons exponentially decay away from the structured interface. Importantly, spoof plasmons can be used to realize relatively large effective modal indices, and as such, can be used to enhance interaction between matter and low-frequency (GHz...THz) electromagnetic waves.

Hybridization of few artificial plasmons has been demonstrated in flat structures [20]. However, the emergence of new electromagnetic waves as result of spoof plasmon hybridization has not been demonstrated so far. Here we show that such hybridization provides a powerful tool for developing GHz-optical mapping that can be used for design and development of practical antenna devices.

Of particular interest to this work are the spoof plasmons supported by the corrugated wires [13], schematically shown in Fig. 2(a). Figure 2(b) shows dispersion of the fundamental spoof plasmon mode in the structure with radii  $r_1 = 15mm$  and  $r_2 = 10mm$  and corrugation period d = 2mm, calculated with finite-element-method (FEM) numerical solver of Maxwell equations [21]. It is seen that the behavior of this mode is similar to the one of the surface plasmon polariton with effective plasma frequency of 12 GHz. Note that, as it is often the case with spoof plasmon structures, the period of corrugation is deeply subwavelength  $(d \ll \lambda_n)$  so that corrugated wire essentially operates in "effective medium" regime [14].



Fig. 2. (a) Schematic geometry of a corrugated wire, (b) dispersion relationship of the spoofplasmon mode supported by the wire

## 4. Spoof wire metamaterials

As described above, the unique optical response of nonlocal hyperbolic nanowire composites can be related to hybridization of natural plasmonic modes. The main hypothesis, to be

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verified in this work is whether hybridized spoof plasmons can be utilized to mimic nonlocality and hyperbolicity at the low frequency (GHz) regime.

Geometry of the proposed metamaterial is schematically shown in Fig. 3(a). The metamaterial represents a collection of aligned corrugated wires with subwavelength corrugation and subwavelength period a (here we use wires described in Fig. 2 arranged in square lattice with a = 50mm). As shown in Fig. 2, individual wires support propagation of spoof plasmon modes.

The modes of the corrugated wire composite are analyzed with commercial finiteelement-method (FEM) solver, COMSOL multiphysics. In these calculations, we solve for the eigen frequency of the modes based on the values of the components of the (quasi-) wavevector  $\vec{k}$ . Figure 3(b...e) represents dispersion and field distribution of the three lowestfrequency modes supported by the metamaterial propagating parallel to its optical axis.



Fig. 3. (a) Schematic geometry of corrugated-wire metamaterial; (b) dispersion of the transverse (blue line) and longitudinal (red line) modes; panels (c,d,e) illustrate the field profiles in TE- polarized, TM-polarized, and longitudinal wave in the composite; compare to Fig. 1

From the homogenized metamaterial perspective [when the fields are averaged over the unit cell], these two of these modes have non-vanishing components of in-plane electric and magnetic fields, while the third mode has non-vanishing averaged  $E_z$  component. Therefore, similar to the case of plasmonic wires (see Fig. 1), these modes can be called ordinary (TE), and extraordinary (TM/longitudinal) waves. Notably, in further similarity to optical response of plasmonic wire metamaterials, two transverse modes of corrugated wire composites have identical dispersion, while dispersion of longitudinal wave resembles that of its smooth wire counterpart.

Dispersion and the field profiles of the three modes was analyzed for a range of in-plane  $(k_x)$  and out-of-plane  $(k_z)$  wavenumbers. These studies show that oblique propagation  $(k_x \neq 0)$  removes degeneracy between the two transverse waves and introduces avoided crossing between the two extraordinary modes (typical dispersion is shown in Fig. 4).

All in all, we note the drastic similarity between the field profiles and dispersions of the modes in spoof-wire systems and the optical behavior of plasmonic smooth-wire composites. These similarities motivate the application of the effective medium theory of nonlocal optical response of nanowire systems [15] to describe low-frequency electromagnetism of spoof wire metamaterials.

# 5. Effective medium description of nonlocal spoof wire composites

Successful application of effective medium theory requires knowledge of the frequencydependent effective medium parameters  $\epsilon_{\perp}, \epsilon_{zz}$ , and  $\xi$ . While in the limit of plasmonic wires these parameters can be directly related to permittivity of metal and host matrix, these expressions cannot be used in the case of corrugated highly-conductive wires. Therefore, in order to test the applicability of the Eqs. (3), we extracted the frequency-dependent effective medium parameters by least-square fitting numerical data to Eqs. (3). Spectral response of effective medium parameters is illustrated in Fig. 4. The same figure presents nonlocal effective medium permittivity of the system.



Fig. 4. Panels (a,b,c) illustrate effective medium parameters of the spoof-wire composite; panel (d) shows dispersion of the TM (solid lines) and TE (dashed line) polarized modes propagating obliquely ( $k_x = \pi a / 2 \approx 0.03 \, mm^{-1}$ ) to the wires; lines represent analytical Eqs. (3), symbols correspond to FEM solutions of Maxwell equations.

Figure 4(d) demonstrate the validity of the proposed effective medium description comparing the predictions of Eq. (3) and exact numerical solutions of Maxwell equations. It is clearly seen that Eq. (3) adequately describes electromagnetism in corrugated wire systems.

Importantly, the corrugated wire composite provides complete map of optics of nonlocal hyperbolic wires to low (GHz) frequencies. The composite supports three waves, comprising one ordinary wave with spherical dispersion and two extraordinary modes, one with elliptical/epsilon-near-zero dispersion and one with hyperbolic-like dispersion.

# 6. Conclusions

To conclude, we demonstrated a new metamaterial based on arrays of corrugated wires that is capable of realizing nonlocal hyperbolic and epsilon-near-zero electromagnetism in lowfrequency systems. We also demonstrated a quantitative map of the electromagnetic properties of such corrugated wire composite on high-frequency (optical) response of plasmonic nanowire composites.

By appropriately scaling geometric parameters of corrugated wire structures, the optical response of the composite can be engineered throughout far-IR...THz...GHz frequency ranges where propagation of spoof plasmons has been already demonstrated in few-interface structures.

Applicability of the effective medium description, proposed in this work is limited to  $|k_x| \lesssim \pi / a$ . The range of applicability of the proposed formalism can be significantly extended by designing metamaterials with deep subwavelength period that can be realized when the space between corrugations is filled with high-index media.

The proposed new class of corrugated wire metamaterials brings new avenues of engineering refractive response and optical density of states to the low-frequency (far IR...GHz) electromagnetic domain. Engineered nonlocality and hyperbolicity can be utilized to optimize detection and generation of low-frequency radiation, high resolution imaging, and enhancement of nonlinerities.

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